At least at least shifts the question[1]

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Odd sentences

- Disjunctions whose disjuncts are in a relation of contextual entailment are typically odd (1). This is Hurford's Constraint (Hurford, 1974). (1a-b) are called **H**urford **D**isjunctions (**HD**).

(1) a. # Ed **studied in Paris** or **in France**. $\mathbf{p}^+ \vee \mathbf{p}$

b. # Ed **studied in France** or **in Paris**. $\mathbf{p} \vee \mathbf{p}^+$

- Related conditionals, dubbed **H**urford **C**onditionals (**HC**, Mandelkern and Romoli, 2018) feature an asymmetry (2).
- This is surprising, because HCs and HDs are isomorphic, assuming material implication and double-negation introduction.

(2) a. # If Ed did**n't** **study in Paris**, he **studied in France**. $\neg\mathbf{p}^+ \rightarrow \mathbf{p}$

b. If Ed **studied in France**, he did**n't** **study in Paris**. $\underbrace{\mathbf{p}}_{\neg(\neg\mathbf{q}^+)} \rightarrow \underbrace{\neg\mathbf{p}^+}_{\mathbf{q}}$

Odd sentences: existing accounts

(1) a. # Ed **studied in Paris** or **in France**.

b. # Ed **studied in France** or **in Paris**.

(2) a. # If Ed did**n't** **study in Paris**, he **studied in France**.

b. If Ed **studied in France**, he did**n't** **study in Paris**.

- Accounting for (1-2) is challenging. Two ways around it:
  - SUPER-REDUNDANCY (Kalomoiros, 2024) gives a special status to overt negation when it comes to evaluating oddness;
  - Compositional Implicit QuDs (Hénot-Mortier, to appear) exploits the notion of specificity.
- In this talk, **I focus on two ways to repair (1-2): at least-insertion, and but-periphrasis.**

Repairing oddness: at least

- *At least* rescues (1a),[1] cf. (3a), but not (1b), cf. (3b). This suggests *at least* needs an antecedent proposition to do its job.

(3) a. Ed **studied in Paris** or **at least** **in France**. $\mathbf{p}^+ \vee \mathrm{AL}(\mathbf{p})$

b. # Ed **at least** **studied in France** or **in Paris**. $\mathrm{AL}(\mathbf{p}) \vee \mathbf{p}^+$

- *At least* also rescues (2a), cf. (4a). But it degrades (2b), cf. (4b). This is consistent with the badness of (3b).

(4) a. If Ed did**n't** **study in Paris**, he **at least** **studied in France**.
$\neg \mathbf{p}^+ \rightarrow \mathrm{AL}(\mathbf{p})$

b. # If Ed **at least** **studied in France**, he did**n't** **study in Paris**.
$\mathrm{AL}(\mathbf{p}) \rightarrow \neg \mathbf{p}^+$

[1] Singh, 2008a; Marty and Romoli, 2022; Zhang, 2022; Krifka, 2024, i.a.

Previous insights on at least in Hurford Disjunctions

- Singh (2008b) suggests *or at least* is corrective, and as such overwrites the disjunctive status of repaired HDs.
- Contra Singh, Zhang (2022) proposes that *at least* operates within an unaltered disjunctive environment, and introduces a more specific QuD, besides the one raised by its prejacent. This way, the two disjuncts of (3a) end up addressing distinct QuDs, and the whole disjunction no longer violates a "QuD-oriented" variant of Hurford's Constraint
- Krifka (2024) also proposes to leave the disjunction unaltered, but assumes each disjunct corresponds to an embedded Speech Act. *At least* then weakens the Speech Acts associated with the second disjunct, s.t. the whole disjunction no longer violates a "Speech Act-oriented" variant of Hurford's Constraint.

Issues with existing accounts

- Existing accounts do not straigthforwardly extend to conditionals:
    - if conditionals are material, the repairing effect of *at least* in the HC (4a) can be captured, but the initial contrast between the repairless HCs (2a-b) is not covered;
    - if conditionals are non-material, the above accounts may cover repairless HCs, but would have to amend their implementation of Hurford's Constraint. (disjunction-specific). Additionally, the amended constraint would have to properly interact with *at least* to capture its repairing effect in (4b) vs. (4a).
- I will show that the Compositional Implicit QuD framework, which directly builds on Zhang's model and accounts for repairless HDs and HCs, extends relatively easily to Hurford Sentences with *at least*, without any revision of the principles modeling Hurford's Constraint in that framework.

Repairing oddness: but

- Contrasting the weaker disjunct with the negation of the stronger one (using *but* as a connective), rescues both HDs in (1), cf. (5). Such structures were dubbed Quasi HDs (Marty & Romoli, 2022).

(5) a. Ed **studied in Paris** or **in France but not Paris**. $\mathbf{p}^+ \vee (\mathbf{p}\ \mathrm{B}\ \neg\mathbf{p}^+)$

b. ? Ed **studied in France but not Paris** or **in Paris**. $(\mathbf{p}\ \mathrm{B}\ \neg\ \mathbf{p}^+) \vee \mathbf{p}^+$

- Zhang proposed an additional constraint on given material to capture Quasi HDs. This is nice, but did not come for free.
- Additionally, *but* does not rescue the HC (2a) (cf. (6a)), and happens to degrade the HC (2b) (cf. (6b))!

(6) a. # If Ed did**n't study in Paris**, he **studied in France but not Paris**. $\neg\mathbf{p}^+ \rightarrow (\mathbf{p}\ \mathrm{B}\ \neg\mathbf{p}^+)$

b. # If Ed **studied in France but not Paris**, he did**n't study in Paris**. $(\mathbf{p}\ \mathrm{B}\ \neg\mathbf{p}^+) \rightarrow \neg\mathbf{p}^+$

Summary of the data, and preview of my predictions

Structure	Ordering	Felicity	My prediction
HD (1)	sw/ws	✗	Redundant
HC (2)	sw	✗	Irrelevant
	ws	✓	
HD+AL (3)	sw	✓	
	ws	✗	Plain odd/Redundant[2]
HC+AL (4)	sw	✓	
	ws	✗	Plain odd/Redundant
HD+But (5)	sw/ws	✓	
HC+But (6)	sw	✗	Irrelevant
	ws	✗	Redundant/Irrelevant

[2] Plain odd if no overt QuD provided (impossible for the sentence to evoke a QuD in the first place); redundant otherwise

The challenges

- We want an implementation of Hurford's Constraint that interacts with binary operators ($\vee/\rightarrow$), and a model of repairs (*at least*, *but*), such that a contrast is derived between:
  - repairless Hurford Disjunctions (always bad) vs. Conditionals (good if **weak**-to-**strong**);
  - the repairing effect of *at least* (always effective if preceded by an antecedent proposition), and *but* (ineffective in conditionals).
- This is a difficult dance: conditionals have to be sufficiently distinct from disjunctions, to derive a contrast between the repairless HCs (2a-2b), and the uselessness of *but* in their repaired counterparts, but also close enough to disjunctions to capture the unified effect of *at least* in HDs (1a) and HCs (2a)!
- We build on the Compositional Implicit QuD framework to achieve this.

General approach of the Compositional Implicit QuD framework

- Add a inquisitive layer to meaning, whereby $\vee$ and $\rightarrow$ differ. This layer does not replace intensional representations, but is intended to be used for pragmatic purposes.
- Implement Hurford's Constraint to be sensitive to inquisitive contributions to meaning. **This has the power to capture a contrast between repairless Hurford Disjunctions and Conditionals.**
- Model *at least* and *but* as operating on inquisitive representations. **This has the power to capture the differential repairing effect of at least and but, on Disjunctions vs. Conditionals.**
- Additional desideratum: the inquisitive layer has to "make sense" beyond its interaction with Hurford phenomena, especially when it comes to the interpretation it assigns to connectives and repair operators.

Roadmap of the talk

In this talk, I will:

1. Sketch how the contrast between repairless Hurford Disjunctions and Conditionals can be captured in the Compositional Implicit QuD framework.
2. Propose and motivate a model of *at least* and *but* at the level of locally accommodated QuDs, susceptible to neutralize the violations of repairless Hurford Sentences.
3. Set out the compositional QuD machinery and show how *at least* and *but* produce modified accommodated QuDs at the sentence level.
4. Spell out two constraints on QuD well-formedness and discuss how they interact with the modified QuDs evoked by repaired Hurford Sentences.

Sketching an account of the contrasts between repairless Hurford Sentence

Recap of the issue and sketch of the solution

- Goal: explain why HDs are bad regardless of the order of the disjuncts[3], while isomorphic HCs exhibit an asymmetry.
- We only sketch the account here (specifics introduced later). Core intuition:
  - **Disjunctions** package information **symmetrically**, making **both disjuncts at issue** at the same time. This causes Hurford Disjunctions to feel **redundant** in both orders.
  - **Conditionals** package information **asymmetrically**, making the **consequent at issue** whenever the antecedent holds. This causes Hurford Conditionals like (2a) to feel incrementally **irrelevant** because the consequent introduces information that is less specific than what the antecedent already introduced as "premise".

[3] Note that this is only the case with non-scalar disjuncts. A separate challenge is to explain how scalar reasoning interacts with Hurford's Constraint. We don't cover this here, but see e.g. Singh, 2008a; Fox and Spector, 2018; Tomioka, 2021; Hénot-Mortier, 2023, 2024; Krifka, 2024.

Core assumptions (to be formalized later in the talk)

- LFs compositionally evoke QuDs in the form of **trees** that organize the worlds of the Context Set in a hierarchical fashion (pretty much like Ippolito (2020)'s Structured Sets of Alternatives).
- In these trees, LFs also **flag** the nodes that entail their propositional content, to signal how the implicit QuD is actually answered.
- $\vee$ and $\rightarrow$ have distinct "inquisitive" contributions; i.e. A $\vee$ B and $\neg$A$\rightarrow$ B evoke different "flagged" QuD trees.
- Hurford's Constraint takes the form of **general-purpose well-formedness principles made sensitive to distinctions in "inquisitive" content**. More specifically:
    - Hurford's Constraint is recast as two constraints (Redundancy and Relevance) on the derivation of QuD trees from a given LF.
    - Oddness arises when such constraints together rule out all the possible QuD trees evoked by a given LF. **"A good sentence has to be a good answer to a good question"** (Katzir & Singh, 2015).

Sketching the issue with repairless Hurford Disjunctions

- The HDs in (1) respectively get paired with the tree in Figure 1, which makes both disjuncts at issue, by flagging the corresponding nodes.

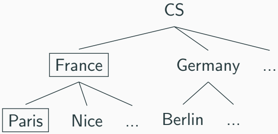


Figure 1: Tree for **Paris** ∨ **France** / **France** ∨ **Paris**.

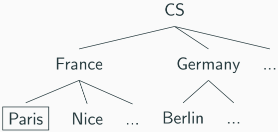


Figure 2: Tree for **Paris**.

- **This tree is "equivalent" to a tree evoked by the Paris-disjunct** (cf. Figure 2). "Equivalent" is taken to mean same tree-structure, and same minimal paths to flagged nodes (note that any path from the root to **Paris**, automatically goes through **France**!). **Tree 1 is thus Q-Redundant.**
- The Hurford Disjunctions in (1) in turn cannot be paired to any well-formed QuD tree, and so are deemed odd.

Taking stock: disjunctive case

- The problem with Hurford Disjunctions, is (very roughly) that they make at issue two nodes (e.g. **Paris** and **France**) that are on the same path on the QuD tree.
- This could be avoided **if France, was understood as a disjunction over French cities** (*Paris or Nice or Lyon or...*): an utterance of **France** would then flag all French cities–instead of **France** as a whole–in the evoked QuD tree (cf. Figure 3).
- Consequently, the nodes flagged by the HDs in (1) would end up being on independent paths.

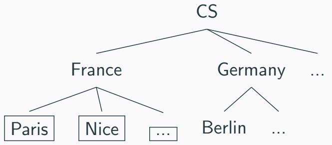


Figure 3: A tree for **France** that could avoid Q-REDUNDANCY in (1).

- We'll propose that *at least* and *but* affect the QuD tree locally evoked by **France** in this way–although some extra complications will arise once we consider a more formal definition of Q-REDUNDANCY.

Sketching the issue with repairless Hurford Conditionals

- The Hurford Conditionals in (2) get paired with the trees in Figures 4-5 (other trees possible, but they don't jeopardize the general result).

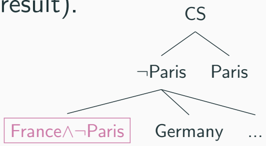


Figure 4: Tree for ¬**Paris**→**France**.

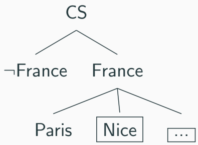


Figure 5: Tree for **France**→¬**Paris**.

- The issue with Tree 4 (which arises with all the trees evoked by (2a)), is that the **France** node originally flagged by the consequent LF, gets shrunk (non trivially intersected with ¬*Paris*) when the tree for the whole conditional gets built. **We assume this shrinkage of flagged nodes constitutes a violation of Q-Relevance.**
- Tree 5 does not have this issue, because, ¬**Paris** can give rise to a partition that is fine-grained enough for the (city-level) flagged nodes to properly fit within the **France**-domain, without shrinkage.

Taking stock: conditional case

- The problem with the Hurford Conditional (2a), is that the consequent introduces a partition that is coarser-grained than the antecedent's–causing the flagged **France**-node to be shrunk when intersected with the ¬**Paris** domain verifying the antecedent.
- This again could be avoided **if France was flagging French cities!** Because then no French city would shrink when fitted into a ¬**Paris** domain: every French city is fully **Paris**, or fully ¬**Paris**.

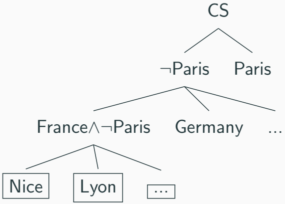


Figure 6: A tree for ¬**Paris**→**France** that would comply with Q-RELEVANCE.

- To derive a difference between *at least* (effective in HCs) and *but* (ineffective/detrimental), while retaining their uniform behavior in HDs, we'll propose that *at least* and *but* both increase specificity, but crucially, **at least erases the at-issueness of its prejacent, while but does not.**

Modeling at least and but

QuD trees

- To determine what kind of QuD *at least* **France** and **France** *but not* **Paris** evoke, one must first properly determine possible QuDs for **Paris**, and **France**.
- We therefore introduce the Compositional Implicit QuD framework in more detail. We take **QuD trees** to be s.t.:
  - All nodes are sets of worlds;
  - The root is the Context set;
  - Any intermediate node is partitioned by the set of its children.

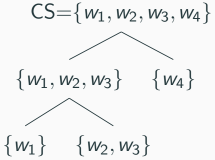


Figure 7: A toy QuD tree.

QuD trees for simplex and negated LFs

- QuD trees evoked by simplex LFs such as *Ed studied in* **Paris**/**France**, are s.t.:
  - Leaves correspond to the partition generated either by the proposition denoted by the LF (*prejacent*), or the prejacent, plus same-granularity alternatives to it. **The leaves entailing/contained in the prejacent are flagged** as true answers.
  - Potential intermediate layers are partitions generated by same-granularity alternatives to a weaker alternative to the prejacent.[4]
- To form QuD tree for negated LFs (e.g. *Ed did not study in* **Paris**), flagged nodes are simply swapped layer-wise.

[4] If there are many such layers, they are ordered according to the strength of the alternative used to generate them: stronger alternatives form lower layers than weaker ones. Forming intermediate layers can lead to branching issues in cases different from **Paris**/**France**. See Appendix for more complete definition handling such cases.

City-specific QuD trees

- In Tree 8a, the leaf layer is generated by **Paris** only.
- In Tree 8b, the leaf layer is generated by city-level alternatives to **Paris**.
- In Tree 8c, the leaf layer is generated by city-level alternatives to **Paris**, and the intermediate layer by country-level alternatives to **France**, which is weaker than **Paris**. Edges are straightforward.
- In all Trees, **Paris** (the prejacent) is flagged.

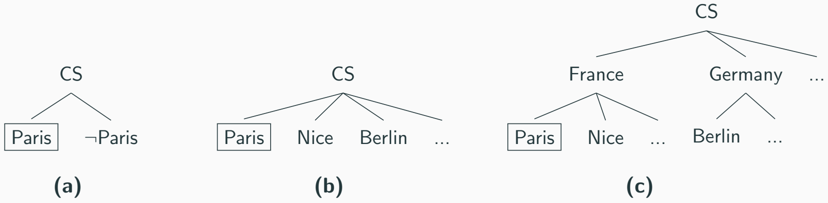


Figure 8: QuD trees evoked by *Ed studied in* **Paris**.

Country-specific QuD trees

- In Tree 9a, the leaf layer is generated by **France** only.
- In Tree 9b, the leaf layer is generated by country-level alternatives to **France**.
- In all Trees, **France** (the prejacent) is flagged.[5]

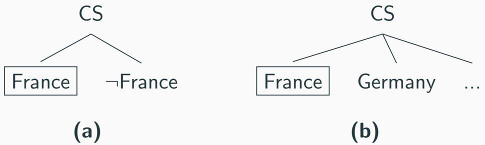


Figure 9: QuD trees evoked by *Ed studied in* **France**.

[5] One could generate QuD trees with coarser-grained intermediate layers, but we omit them here to remain symmetric with the **Paris**-case.

The licensing and effects of at least

- When is *at least p* licensed, and what does it do to the QuD(s) evoked by its prejacent?
- (7a) shows that *at least* is bad in out-of-the-blue matrix sentences, or when there is an overt QuD that is either not relevant, or s.t. its maximal answer at most as specific as *at least*'s prejacent.
- *At least* is good when **a QuD more specific than at least's prejacent is retrievable.**
  - This QuD may be overt (7b), or evoked by a sentence preceding *at least*, e.g. *Ed studied in* **Paris** in (7c).
  - This also suggests that **at least makes the QuD evoked by its prejacent more specific.**

(7) a. Jo: .../Is Ed happy today?/In which **country** did Ed study?
Al: # Ed studied at least in {**France**/**Paris**}.

b. Jo: Which is the **city** where Ed studied?
Al: Ed studied at least in {**France**/#**Paris**}.

c. Al: Ed studied in **Paris**, or at least in **France**.

Modeling at least: general case

- We model the QuD evoked by *at least* X in the following way:
  - it is **defined only if a antecedent QuD (overt or evoked by preceding material) is available**. The tree associated with this QuD has to strictly contain a tree evoked by X (in terms of nodes, and edges).
  - if defined, it is structurally equal to a tree of the antecedent QuD verifying the containment precondition. Flagged nodes are leaves entailing the proposition denoted by X (*at least*'s prejacent).

Modeling Ed studied at least in **France**

- In HDs and HCs, a QuD tree evoked by *at least* **France**, will be defined if there is an overt QuD that is e.g. about cities, or, if *at least* **France** is preceded by a parallel proposition (1st disjunct/antecedent) about e.g. **Paris**.

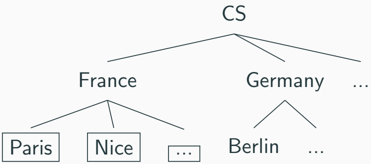


Figure 10: QuD tree evoked by *at least* **France**, if an antecedent (city-level) QuD is retrievable.

- If defined, **this QuD tree will be structurally equal to a QuD tree for Paris, but will have all French cities flagged as true answers.** Cf. Figure 10.
- Note that this definition **totally erases the at-issueness of France.** This is justified by (8).

(8) Jo: In which **city** did Ed study? I don't care which **country**!
Al: Ed studied at least in **France**.

Connection to previous accounts of at least

- We just modeled *at least* in the Implicit QuD framework, but connections can be established between this model and the idea of Speech Act weakening.
- In our model, **at least shifts the question to something more specific** than what was originally raised by its prejacent.
- Nevertheless, **at least somehow retains the intensional message of its prejacent**: unioning the flagged nodes of *at least*'s output QuD tree (e.g. French cities), yields a proposition that is equal to *at least*'s prejacent (e.g. **France**).
- This interaction between the message and its packaging may be reinterpreted as Speech Act weakening: the propositional content of *at least p* and *p* is the same, but **because at least increases the specificity of the implicit QuD, this content also becomes less determinate in the case of at least p**: it gets split across different nodes of the QuD tree, pretty much as if *at least p* was disjunctive.

The licensing and effects of but

- *But* is a contrastive operator that can give rise to (asymmetric) Hurford-like effects (Tomioka, 2021).

(9) a. # Ed studied in **PARIS** but Al studied in **FRANCE**.
    b. Ed studied in **FRANCE** but Al studied in **PARIS**.

- Although I don't have a full account of (9), these examples suggest that *but* is **sensitive to the relative degrees of specificity** conveyed by its two arguments (just like HCs[6]), but also, make its **two arguments at-issue** (just like HDs).

[6] It's worth noting that *but* could be replaced by *if* in (9), while retaining more or less the same meaning.

The licensing and effects of but

- What does *but* do to the QuD(s) evoked by its two arguments?
- *But* is ok out-of-the-blue, and can answer questions about its first argument, when the second argument is also somehow important.[7]

(10) a. Jo: In which **country** did Ed study? # I don't care which **city**.
Al: Ed studied in **France** but not **Paris**

b. Jo: In which **city** #(or **country**) did Ed study?
Al: Ed studied in **France** but not **Paris**

[7] The 1st argument still has some extra prominence as opposed to the 2nd, because any overt question answered by *but has* to mention the level of specificity of *but*'s 1st argument (cf. (10b)), even if the 2nd argument is more specific (and therefore settling it settles everything). We abstract over any difference in prominence here, and simply treat both arguments of *but* as at-issue.

Modeling but: general case

- We model the QuD evoked by *X but Y* in the following way:
    - Build a QuD tree for X and replace all its flagged nodes by their intersection with a QuD tree for Y.
    - The intersection between a tree *T* and a node *N*, is *T* whose nodes are each intersected with *N* (and empty nodes and trivial edges are removed).
    - Retain the flagged nodes of *both* X's and Y's QuD trees.
- We will see that this building process:
    - retains the **structural** aspects of the building of **conditional** QuD trees (in that a QuD tree for the 2nd argument is "plugged" into a QuD tree for the 1st)...
    - and the **"flagging"** aspects of the building of **disjunctive** QuD trees (in that both arguments are made at-issue).
- In other words, *but* is "inquisitively" like **a conditional that assigns equal at-issueness to both it arguments.**

Modeling Ed studied in France but not Paris

- A QuD tree for **France** *but not* **Paris**, is a QuD tree for **France** (as in Figure 9), where the **France**-node is replaced by its intersection with a QuD tree for ¬**Paris**.
- **France** and city nodes different from **Paris** are flagged. This is all done in Figure 11.

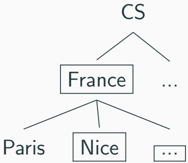


Figure 11: QuD tree evoked by **France** *but not* **Paris**.

- Other trees are also possible, because **France** and **Paris** are themselves compatible with multiple trees. We omit these extra trees here because they don't jeopardize the final result.
- Contrary to the *at least* case, **the at-issueness of France is retained.**

- Now that we have locally derived QuD trees for the repaired fragments of the sentences at stake, we need to compute the effects of these repairs at the global level.

Building up repaired QuD trees

Next steps

- We have a model of the local QuDs evoked by *at least* and *but* in sentences like (3), (4), (5), and (6).

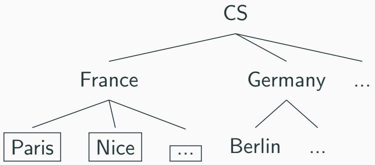


Figure 12: QuD tree evoked by *at least* **France**, if an antecedent (city-level) QuD is retrievable.

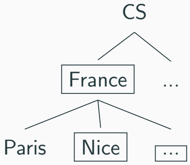
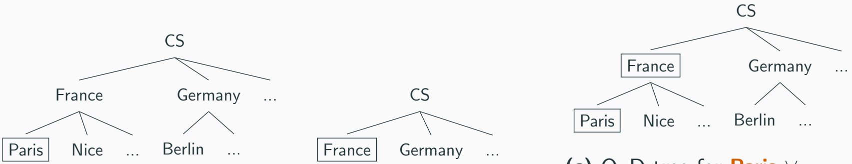


Figure 13: QuD tree evoked by **France** *but not* **Paris**.

- We now need to **compute the effects of these operators at the sentence level**, and determine when the resulting "repaired" QuD trees escape our specific implementation of Hurford's Constraint.
- We'll start with disjunctions, and then deal with conditionals. To this end, we will **flesh out rules to build disjunctive and conditional QuD trees.**

Building repairless HDs (1)

- To build **disjunctive QuD trees** for LFs of the form $X \vee Y$, one just builds all the possible **unions** of QuD trees for X and for Y, and retains the ones that are well-formed. Flagged nodes are **inherited** from the two unioned QuD trees.
- The only possible QuD tree derived from the repairless HDs in (1), is given below, along with the QuD trees used to build it.

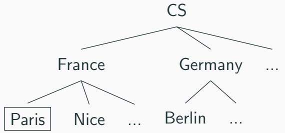


(a) QuD tree for **Paris**. **(b)** QuD tree for **France**. **(c)** QuD tree for **Paris** $\vee$ **France** / **France** $\vee$ **Paris**.

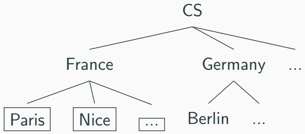
Figure 14: Deriving the QuD tree evoked by (1a)=#*Ed studied in* **Paris** *or* **France**, or (1b)=#*Ed studied in* **France** *or* **Paris**.

Building HDs repaired by at least (3a)

- The effect of *at least* in the HD (3a) is computed below (the Appendix deals with (3b)).



(a) QuD tree for **Paris**.



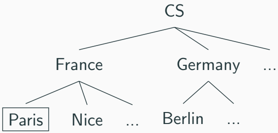
(b) QuD tree for AL(**France**) (if an antecedent city-level QuD is retrievable). Also QuD tree for **Paris** $\vee$ AL(**France**).

Figure 15: Deriving the QuD tree evoked by (3a)=*Ed studied in* **Paris** *or at least* **France**.

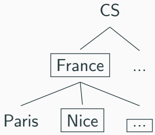
- The fact that *at least* **France**, and **Paris** *or at least* **France**, give rise to the same QuD tree, may seem problematic, in terms of redundancy. We'll discuss why it may not be in the last section.

Building HDs repaired by but (5)

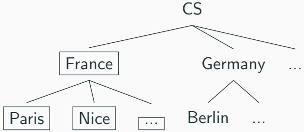
- The effect of *but* in (5a-b) is computed below.[8]
- The only difference with *at least*-repairs in HDs, is that **France** remains flagged in the case of *but*-repairs. We'll see that this extra flagged node does not have any consequence in HDs, in terms of redundancy.



(a) QuD tree for **Paris**.



(b) QuD tree for **France** B ¬**Paris**.



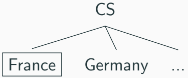
(c) QuD tree for **Paris** ∨ **France** B ¬**Paris** / (**France** B ¬**Paris**) ∨ **Paris**.

Figure 16: Deriving the QuD tree evoked by (5a)=*Ed studied in* **Paris** *or* **France** *but not* **Paris**, or (5b)=*Ed studied in* **France** *but not* **Paris**, *or in* **Paris**.

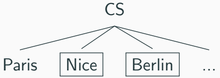
[8] Even if we omitted some possible QuD trees for **France** *but not* **Paris**, such trees are filtered out when building a QuD tree for the whole disjunction, as done in the above Figure.

Building repairless HCs: felicitous case (2b)

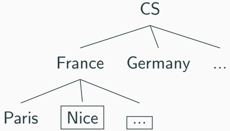
- To build **conditional QuD trees** for LFs of the form $X \rightarrow Y$, one takes a QuD tree for X and replaces each of its flagged nodes, by its intersection with a QuD tree for Y. Flagged nodes are **inherited** from the QuD tree evoked by Y that was used to perform the intersection operation.
- A possible QuD tree for the felicitous repairless HC (2b), is given below, along with the QuD trees used to build it. Other trees are possible, but don't jeopardize the general result.



(a) QuD tree for **France**.



(b) QuD tree for ¬**Paris**.

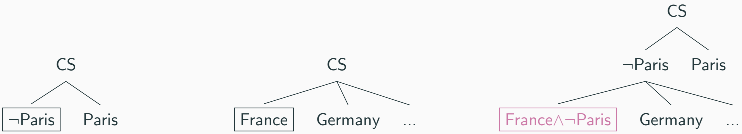


(c) QuD tree for **France**→¬**Paris**.

Figure 17: Deriving the QuD tree evoked by (2b)=*If Ed studied in* **France**, *he did not study in* **Paris**.

Building repairless HCs: infelicitous case (2a)

- A possible QuD tree for the infelicitous repairless HC (2a), is given below, along with the QuD trees used to build it.
- Other trees are possible, but don't jeopardize the general result (i.e. end up all equally problematic, due to a shrinkage of the flagged **France**-node).



(a) QuD tree for ¬**Paris**.

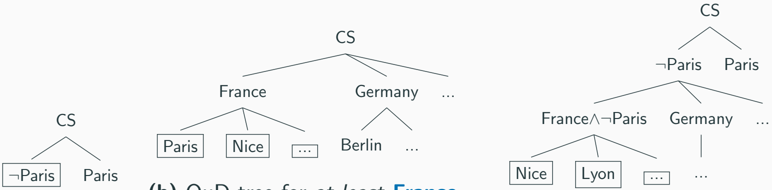
(b) QuD tree for **France**.

(c) QuD tree for ¬**Paris**→**France**.

Figure 18: Deriving the QuD tree evoked by (2a)=#*If Ed did not study in* **Paris**, *he studied in* **France**.

Building HCs repaired by at least (4a)

- The effect of *at least* in the HC (4a) is computed below (more trees are possible but this one suffices to make the point).
- Note that the **France**-node still gets shrunk, but is no longer flagged. The case of (4b) is covered in the Appendix.



(a) QuD tree for *not* **Paris**.

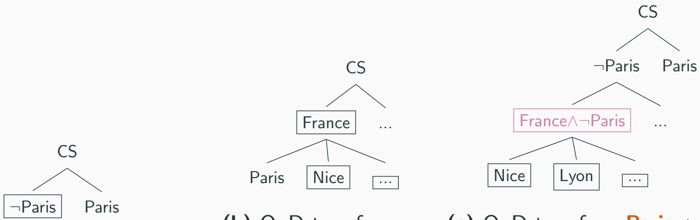
(b) QuD tree for *at least* **France** (if an antecedent city-level QuD is retrievable).

(c) QuD tree for ¬**Paris**→AL(**France**).

Figure 19: Deriving the QuD tree evoked by (4a)=*If Ed did not study in* **Paris**, *he at least studied in* **France**.

Building HCs tentatively repaired by but: unrescuable case (6a)

- The effect of *but* in (6a) is computed below.
- Other trees are possible, but don't jeopardize the general result (i.e. end up all equally problematic, due to a shrinkage of the flagged **France**-node).



(a) QuD tree for ¬**Paris**.

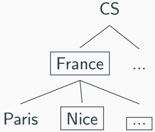
(b) QuD tree for **France** *but not* **Paris**.

(c) QuD tree for ¬**Paris**→ (**France** B ¬**Paris**).

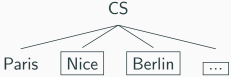
Figure 20: Deriving the QuD tree evoked by (6a)=#*If Ed did not study in* **Paris**, *he studied in* **France** *but not* **Paris**.

Building HCs tentatively repaired by but: degraded case (6b)

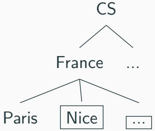
- The effect of *but* in (6b) is computed below. The tree obtained by this process ends up being equal to a tree evoked by the repairless simplification of (6b), (2b).
- The effect of *but* is obfuscated, due to the fact **France** is flagged in the antecedent QuD tree, and so gets *replaced* by its intersection with a consequent QuD tree when the conditions tree gets built. This removes the extra city-partitioning and flagging locally introduced by *but*. Other possible trees don't change this result.



(a) QuD tree for **France** *but not* **Paris**.



(b) QuD tree for ¬**Paris**.



(c) QuD tree for (**France** B ¬**Paris**)→¬**Paris**.

Figure 21: Deriving the QuD tree evoked by (6b)=#*If Ed studied in* **France** *but not* **Paris**, *he did not study in* **Paris**.

Repaired QuD trees, and Hurford's Constraint

Reframing Hurford's Constraint

- As foreshadowed in the first section, we want to replace Hurford's Constraint by **more general-purpose principles, that are sensitive to the relation between LFs and their implicit QuD trees.** 2 principles:
  - Q-Redundancy (building on Katzir and Singh (2014)): don't map an LF to a QuD, if a simpler LF could evoke a similar QuD (same structure, same minimal set of paths to flagged nodes).
  - Q-Relevance (loosely building on Lewis (1988)): don't map an LF to a QuD, if this QuD shrinks nodes flagged by more local QuDs used to compute it. Rationale: whatever is made incrementally at-issue (flagged), has to properly "fit" within the partitioning of the Context Set that is already in place.
- The Appendix further formalizes these principles.
- We've already sketched how these they get checked on repairless Hurford Sentences, so let's now focus on the repaired versions.

“At least” rescues HDs modulo assumptions on QuD-retrieval

- Figure 22 repeats the QuD tree derived for the HD in (3a).
- This tree does not violate Q-RELEVANCE, because no node got shrunk throughout its derivation.
- However, it’s identical to the one evoked by *Ed studied at least in* **France** (a simplification of (3a)), at least **if a suitable antecedent QuD is retrievable in the context of this simplification.**
- We think this retrievability precondition ensures **Q-Redundancy does not kick in**: as soon as (3a) gets simplified into *at least* **France**, *at least* “loses” it antecedent QuD (conundrum of overt QuDs discussed in Appendix).

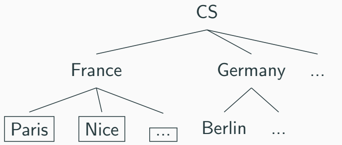


Figure 22: QuD tree for (3a)=*Ed studied in* **Paris** *or at least* **France**.

- Therefore, *at least* **France**, *if simplified from (3a)*, does not evoke any QuD, and so, does not properly compete with (3a) in terms of evoked QuD trees. (3a) is correctly rescued from oddness.

"At least" rescues infelicitous HCs

- Figure 23 repeats the QuD tree derived for the repaired HC (4a).
- This tree does not violate Q-Relevance, because by the time the conditional QuD tree gets built and shrinks the **France** node, this node is no longer flagged: *at least* erased its at-issueness.
- It's also easy to see that this tree is strictly more complex than any QuD tree evoked by a simplification of (4a). So Q-Redundancy is satisfied.

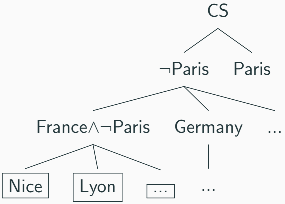


Figure 23: QuD tree for (4a)=*If Ed did not study in* ***Paris****, he at least studied in* ***France***.

- (4a) is thus correctly rescued from oddness.
- The fact the originally felicitous HC (2b) gets bad with *at least* in its antecedent just comes from the fact *at least* lacks an antecedent QuD in that case.

“But” rescues HDs

- Figure 24 repeats the QuD tree derived for the repaired HDs in (5).
- It’s easy to see this tree does not violate Q-RELEVANCE, because no node got shrunk throughout its derivation.
- And it’s not Q-REDUNDANT either. For it to be Q-REDUNDANT given (5a)/(5b), we’d need to find a simplification of (5a)/(5b) leading to the same tree structure and, more importantly, same minimal paths to flagged nodes.

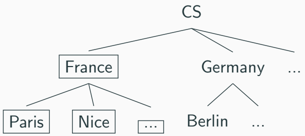


Figure 24: QuD tree for (5a)=*Ed studied in* **Paris** *or* **France** *but not* **Paris**, or (5b)=*Ed studied in* **France** *but not* **Paris** *or* **Paris**

- This means that a simplification of (5a)/(5b) should flag all the French city nodes. Neither **Paris**, **France**, or **France** *but not* **Paris**, achieve this.
- Both repaired HDs in (5) are therefore rescued from oddness.

“But” does not rescue infelicitous HCs

- Figure 25 repeats the QuD tree derived for the HC in (6a).
- Other trees are possible for this sentence, but they will always be evoked by the repairless counterpart of (6a), (2a).
- This is because *but* in the antecedent, preserves the at-issueness of **France**, and so causes a replacement of this node by its intersection with a QuD tree for the consequent (¬**Paris**), when the conditional QuD tree gets built.

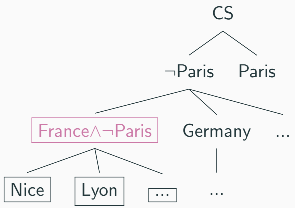


Figure 25: QuD tree for (6a)=#*If Ed did not study in* ***Paris****, he studied in* ***France*** *but not* ***Paris***.

- In other words, whatever *but* added beyond **France** at the level of local QuD trees, gets overwritten. In particular, the inquisitive contribution of its second argument is totally ignored.
- (6b) is therefore correctly predicted to be odd.

"But" degrades otherwise felicitous HCs

- Figure 24 repeats the QuD tree derived for the HD in (6b).
- As evoked by (6b), this tree is Q-REDUNDANT, because it is equal to a tree evoked by the repairless simplification of (6b), (2b).
- (6b) is thus correctly predicted to be odd.

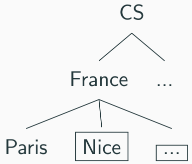


Figure 26: QuD tree evoked by (5b)=#*If Ed studied in* **France** *but not* **Paris**, *he did not study in* **Paris**.

Concluding remarks

The ground covered

- The Compositional Implicit QuD framework provides a **general approach to tackle Hurford Phenomena**, including Hurford Conditionals (Hénot-Mortier, to appear).
- And we saw today that, combined with a relatively sensible model of repairing operators (*at least*, and *but*), it also covers the **intricate felicity profiles of repaired (or tentatively repaired) Hurford Disjunctions and Conditionals**.
- The key aspect of the account that allowed to derive the target contrasts was **"flagging", or at-issueness**: by arguing that *at least* fully shifts the question and erases the at-issueness of its prejacent, while *but* only partially does so, and crucially preserves the at-issueness of its first argument, we could derive that **at least had overall a higher repairing power than but** (*modulo* left-to-right effects).
- We mostly covered out-of-the-blue sentence here, which might seem a bit unrealistic. The Appendix discusses the effect of an overt QuD, especially w.r.t. *at least*.

Remaining puzzles

- There is a variety of cases (pointed out by Krifka, 2024), that the current account does not yet cover.
  - For instance, our account intuitively assigns the same inquisitive contribution to *at least* **France** and *a city in* **France**. But the later expression does not make a good repair, cf. (11)!
  - The effect of *either ... or* in canceling the repairing effect of *at least* (but not *but*!), remains mysterious (cf. (12)).
  - Lastly, our account is currently silent regarding other particles, such as *even*, *maybe*, which also have a repairing effect.

(11) # Ed was born in **Paris** or in a city in **France**

(12) a. # **Either** Ed lives in **Paris**, **or** at least he lives somewhere in **France**.

b. **Either** Ed lives in **Paris**, **or** he lives in **France** but not **Paris**.

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Appendix

Branching issues with intermediate layers

- We defined QuD trees evoked by simplex LFs as having potentially more than one layer. But we did not define how the branching between layers of nodes is achieved, and crucially, if this branching preserves tree-structure!
- No problem arises when considering country- and city-layers: every city clearly belongs to one single country, to the mapping between the city- and country-layers does not create cycles.
- But a problem arises if we consider an intermediate layer generated by $\exists$ ($\{\exists, \neg\exists\}$), and a leaf layer generated by $\forall$ ($\{\forall, \neg\forall\}$). This is because, $\neg\forall$ is compatible with both $\exists$ and $\neg\exists$! So a leaf would end up having 2 possible parents, creating a cycle.
- To overcome this issue, after creating all layers, definitive layers are obtained by dynamically intersecting each layer with the immediately higher one, top down. The problematic leaf layer then becomes s.t. each node has a single parent (either $\forall$ or $\neg\forall$): $\{\exists, \neg\exists\} \cap \{\forall, \neg\forall\} = \{\exists \wedge \forall, \neg\exists \wedge \forall, \exists \wedge \neg\forall, \neg\exists \wedge \neg\forall\} = \{\forall, \exists \wedge \neg\forall, \neg\exists\}$

A note on corrective at least

- The use of *at least* in HDs and HCs may feel "corrective". Zhang (2022) puts forth arguments against this view in English.
- I'd like to concur and discuss a potential argument from French, where corrective and non-corrective uses of *at least* may be teased apart by the lexicon.
  - In addition to *au moins* (lit. *at least*), French displays other strategies to achieve similar effects: *(tout)* **du** *moins* and *(tout)* **au** *moins*.
  - (13) shows that the former seems purely corrective (it cannot be used in simplex answers), while the latter does not, and still has a repairing effect in HDs and HCs (however, it might still be ambiguous, despite the existence of *tout du moins*/*au moins*...).

(13) a. Dans quelle **ville** Ed a étudié?
–Il a (tout) au/#du moins étudié en **France**.

b. Ed a étudié à **Paris**, ou (tout) au/du moins en **France**.

c. Si Ed n'a pas étudié à **Paris**, il a (tout) au/du moins étudié en **France**.

Deriving the left-right asymmetry of at least repairs (3b)/(4b)

(3b) # Ed **at least** **studied in France** or **in Paris**. AL($\mathbf{p}$) $\vee$ $\mathbf{p}^+$

(4b) # If Ed **at least** **studied in France**, he did**n't** **study in Paris**.
AL($\mathbf{p}$) $\rightarrow$ $\neg\mathbf{p}^+$

- Note that the HD (3b), where *at least* occurs in the 1st disjunct, cannot give rise to any QuD out-the-blue, because *at least* lacks an antecedent QuD to do its job in that case. (3b) is thus correctly predicted to be odd out-of-the-blue.
- This extends to cases where an overt QuD is retrievable. In that case, (3b) is Q-REDUNDANT, given its simplification *at least* **France**, which gives rise to the same QuD tree.
- And this extends to the HC (4b), where *at least* occurs in the antecedent. Either *at least* cannot give rise to a QuD tree and we get oddness "for free" (out-of-the-blue case), or it can, but ultimately produces a QuD tree that makes (4b) Q-REDUNDANT given its repairless simplification (2b).

Issues with HDs repaired by at least, given an overt QuD

- We have argued that (3a) could be rescued by *at least*, because, even though it gives rise to the same QuD tree as its simplification *at least* **France**, competition between the two forms and their QuD trees does not take place.
- This is because such a simplification can be said to lack an antecedent QuD, and so cannot itself evoke a QuD. It is therefore not a valid competitor. This **relies on the assumption that QuD dependencies are resolved post-simplification.**
- But what if an overt QuD e.g. *in which* **city** *did Ed study?* is independently provided? (3a) still seems fine (cf. 14).

(14) Jo: In which **city** did Ed study?
     Al: Ed studied in **Paris**, or at least **France**.

- In that context, we'd predict the *at least*-simplification of (3a) to be a valid competitor when evaluating Q-REDUNDANCY, and so (3a) should be odd... the only way around it is to assume **simplifications are also blind to contextually-provided QuDs!**

Unpacking Q-Redundancy

(15) **Q-Redundancy**: LF X is Q-REDUNDANT iff there is a formal simplification X' of X obtained *via* constituent-to-subconstituent substitution, s.t. Qtrees(X) $\leqq$ Qtrees(X').

(16) **Equivalent Sets of Qtrees**: $S \leqq S'$ iff $\forall T \in S.\ \exists T' \in S'.\ T \equiv T'$ (note: it is an asymmetric relation!)

(17) **Equivalent Qtrees**: $T \equiv T'$ iff T and T' have same structure and same set of maximal verifying paths.

(18) **Verifying paths**: set of paths (=ordered list of nodes) from the root to each flagged node.

(19) **Path containment**: $p \subseteq p'$ iff p is a prefix of p'.

(20) **Maximal Verifying Paths (P^*)**: if P is a set of verifying paths, P^* is the set of maximal elements of P w.r.t. path containment.

Rephrasing Relevance

- Under the partition-based view of questions, a proposition p is relevant given a question, if it **does not cut across cells**. We want some generalization of this to apply as a filter during Q-tree derivation.

(21) Q-Relevance: If T'' is derived from T and T' *via* Q-tree composition, then $\mathbb{N}^+_{T''} \subseteq \mathbb{N}^+_T \cup \mathbb{N}^+_{T'}$.

- This means that verifying nodes coming from the Q-trees passed as input to a binary Q-tree composition rule should be either fully ruled-out, or fully preserved in the output Q-tree, i.e., they should not be cut-across.
- A correlate in our $\{\neg, \vee, \rightarrow\}$-fragment (trust me on the meaning of $\vee$ at that point):

(22) Q-Relevance (correlate): If tree T gets intersected with node N, $\mathbb{N}^+_{T \cap N} \subseteq \mathbb{N}^+_T$